

An explicit cubic homogeneous counterexample to the Jacobian Conjecture in dimension 19

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Preliminary draft; comments and corrections welcome

Abstract

Starting from the factor-aware 11-dimensional degree-three reduction of the recently announced Alpöge–Fable map posted in Joe Atkins-Turkish’s Gist, we apply a sparse version of the classical Segre–Yagzhev homogenization procedure. We obtain an explicit map

$$\widehat{G} = X + H : \mathbb{C}^{19} \longrightarrow \mathbb{C}^{19}$$

for which every nonzero component of H is homogeneous of degree three,

$$(\mathbf{J} H)^{18} = 0 \quad \text{and} \quad (\mathbf{J} H)^{17} \neq 0,$$

and three distinct rational points share a common image. Hence $\det \mathbf{J} \widehat{G} \equiv 1$ while $\widehat{G}$ is not injective. The final object is checked twice in exact arithmetic: a SymPy program derives it from the 11-variable map, and a second verifier uses only the Python standard library and a canonical machine-readable statement of the map. Among the timestamped public constructions we located as of 21 July 2026, this reduces the smallest dimension of a fully explicit counterexample in cubic homogeneous Yagzhev form from 79 to 19.

1 Introduction

The Jacobian Conjecture asserts that a polynomial map $F : \mathbb{C}^n \rightarrow \mathbb{C}^n$ with nonzero constant Jacobian determinant is a polynomial automorphism. On 20 July 2026 UTC, Levent Alpöge publicly posted the map [1]

$$F(x, y, z) = \begin{pmatrix} (1 + xy)^3 z + y^2(1 + xy)(4 + 3xy), \\ y + 3x(1 + xy)^2 z + 3xy^2(4 + 3xy), \\ 2x - 3x^2 y - x^3 z \end{pmatrix}, \quad (1)$$

with

$$\det \mathbf{J} F = -2$$

and

$$F(0, 0, -1/4) = F(1, -3/2, 13/2) = F(-1, 3/2, 13/2) = (-1/4, 0, 0).$$

The announcement credited Fable 5 with producing the example [1]. We use the shorthand “Alpöge–Fable map” for this public presentation without attempting to infer a more detailed division of authorship.

The coordinate formula in (1) has since been explained geometrically. In the symmetric-product description of Jiang and the factorization–resultant description of Lou, synthesized and independently

audited by Naskrecki [2, 3, 4], the map forgets a marked simple root of a binary cubic. A generic cubic has three simple roots, which conceptually accounts for its generic three-point fibers. In factor coordinates the underlying operation is the bilinear multiplication $(\ell, q) \mapsto \ell q$; the comparatively high degree of (1) results from the global affine coordinates used to identify both source and target with $\mathbb{C}^3$. This geometry is not needed for the proof below, but it clarifies the origin of the starting map and may be useful for future dimension reduction.

A *Yagzhev map* is a polynomial map $X + H$ in which every nonzero component of H is homogeneous of degree three. The reduction theorems of Yagzhev and Bass–Connell–Wright show that the general Jacobian Conjecture reduces to this class, and Drużkowski further reduced it to cubic-linear form [11, 8, 10]. Thus the three-dimensional counterexample guarantees the existence of a noninjective Yagzhev map in some higher dimension. Our purpose is to make one such map explicit in dimension 19 and to give compact exact certificates for it. The new ingredient of this note is the support-aware homogenization in Proposition 1 and its application to the seven-component cubic support of the normalized 11-variable map.

Public constructions and scope of the contribution

The following table separates the original counterexample, intermediate degree-three reductions, and final Yagzhev-form maps. It is a record of public constructions known to us on 21 July 2026, not a claim of exhaustive priority.

Contributor	Public object	Dimension
Alpöge / Fable 5 [1]	original Keller counterexample	3
Ali Demirci [5]	generic explicit Yagzhev realization	79
u/Distinct-Soft-3991 [6]	degree-three Keller map	17
Joe Atkins-Turkish [7]	posted factor-aware degree-three Keller map; Gist credits ChatGPT	11
Present note	sparse Yagzhev lift with global symbolic certificate	19

Demirci’s script [5] applies 18 monomial degree-lowering gadgets, reaching dimension 39, and then introduces a full set of 39 auxiliary coordinates together with one Segre variable, giving $2 \cdot 39 + 1 = 79$. The present construction instead starts from the 11-variable map posted by Atkins-Turkish [7] and introduces auxiliary coordinates only for the seven components whose normalized cubic part is nonzero.

2 The 11-variable degree-three map

Let

$$X = (x, y, z, a, b, c, d, q, s, h, k).$$

The factor-aware map $\Phi : \mathbb{C}^{11} \rightarrow \mathbb{C}^{11}$ posted by Joe Atkins-Turkish is displayed in Appendix A [7]. It has total degree three and sends the three rational points

$$\begin{aligned} p_1 &= (0, 0, -1/4, 0, 0, 1/2, 0, 0, 0, 0, 0), \\ p_2 &= (1, -3/2, 13/2, -9/4, 9/2, -10, -3/2, 3/4, -153/8, 1, -13/2), \\ p_3 &= (-1, 3/2, 13/2, -9/4, -9/2, -10, -3/2, -3/4, -153/8, 1, 13/2) \end{aligned}$$

to $(-1/4, 0, \dots, 0)$. The accompanying Gist records a derivation of Φ from (1) by explicit stable degree-reduction operations and obtains $\det J\Phi = -2$; it states that the explicit construction, simplification, and verification code were generated by ChatGPT [7]. For the final theorem below, however, no trust in that derivation or determinant calculation is required: the 19-dimensional map will be certified directly.

Let $L = J\Phi(0)$. Explicitly,

$$L(X) = (z, y, 2x, a, b, c + 2z, d, q, s, h, k), \quad \det L = -2.$$

Define

$$\Phi' = L^{-1} \circ \Phi = X + Q + C,$$

where Q and C are respectively quadratic and cubic homogeneous. Their nonzero components are listed in Appendix B. The cubic support is

$$S = \{1, 2, 3, 4, 5, 6, 9\}. \quad (2)$$

Thus only seven auxiliary coordinates are needed in the homogenization step.

3 Sparse Segre–Yagzhev homogenization

The usual construction introduces one auxiliary coordinate for every component. Zero cubic components need none.

Proposition 1 (Sparse Segre–Yagzhev lift). *Let*

$$\Psi = X + Q + C : \mathbb{C}^n \rightarrow \mathbb{C}^n,$$

where Q and C are homogeneous of degrees two and three, respectively. Let

$$S = \{i : C_i \neq 0\}, \quad r = |S|,$$

and write $C = \iota(C_S)$, where $C_S : \mathbb{C}^n \rightarrow \mathbb{C}^r$ collects the components indexed by S and $\iota : \mathbb{C}^r \hookrightarrow \mathbb{C}^n$ inserts zeros in the other coordinates. Define

$$\widehat{\Psi}(X, Y, t) = (X + tQ(X) - t^2\iota(Y), Y + C_S(X), t). \quad (3)$$

Then $\widehat{\Psi} : \mathbb{C}^{n+r+1} \rightarrow \mathbb{C}^{n+r+1}$ has identity linear part and every nonlinear component is homogeneous of degree three. In the terminology of Campbell [9, Theorem 5], it is obtained from Ψ by a Segre equivalence, followed by a stable extension and left–right composition with polynomial automorphisms. Consequently, the Keller property and injectivity are preserved. Any collision $\Psi(P) = \Psi(P')$ lifts at $t = 1$ to

$$(P, -C_S(P), 1), \quad (P', -C_S(P'), 1).$$

Proof. The Segre map associated with Ψ is

$$T(X, t) = (t^{-1}\Psi(tX), t) = (X + tQ(X) + t^2C(X), t),$$

where the second expression gives its polynomial extension at $t = 0$. Its X -Jacobian block is $J\Psi(tX)$, and the last coordinate is t , so

$$\det JT(X, t) = \det J\Psi(tX).$$

Moreover, T is injective if and only if Ψ is injective: equality of two T -images first gives equality of the t -coordinates; at $t = 0$ the first coordinate is X , while at $t \neq 0$ injectivity reduces to that of Ψ after scaling by t . Conversely, every collision of Ψ appears in the slice $t = 1$. This is the Segre-equivalence step.

Now stabilize T by the Y -coordinates and write

$$S_0(X, Y, t) = (X + tQ(X) + t^2C(X), Y, t).$$

Define triangular automorphisms

$$A_2(X, Y, t) = (X, Y + C_S(X), t), \quad A_1(X, Y, t) = (X - t^2\iota(Y), Y, t).$$

Since $C = \iota(C_S)$, direct substitution gives

$$A_1 \circ S_0 \circ A_2 = (X + tQ(X) - t^2\iota(Y), Y + C_S(X), t),$$

which is (3). Stable extension and left–right composition with polynomial automorphisms preserve the Keller property and injectivity. At $t = 1$, substituting $Y = -C_S(P)$ gives first coordinate $P + Q(P) + C(P) = \Psi(P)$ and second coordinate zero, proving the collision statement. $\square$

Lemma 1. *Let $G = X + H$ over $\mathbb{Q}$, and suppose JH is nilpotent. Then $\det JG \equiv 1$.*

Proof. If $(JH)^m = 0$, then

$$(I + JH)^{-1} = I - JH + (JH)^2 - \cdots + (-1)^{m-1}(JH)^{m-1}.$$

Thus $\det(I + JH)$ is a unit in the polynomial ring and hence a nonzero rational constant. At every specialization, JH remains nilpotent, so $\det(I + JH) = 1$. $\square$

4 The 19-dimensional map

Apply Proposition 1 to Φ' with the support (2). Let

$$Y = (Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_9)$$

and define

$$\widehat{G}(X, Y, t) = (X + tQ(X) - t^2\iota(Y), Y + C_S(X), t). \quad (4)$$

This is a polynomial self-map of

$$\mathbb{C}^{11+7+1} = \mathbb{C}^{19}.$$

Together with the formulas for Q and C in Appendix B, equation (4) is a complete explicit specification of the map.

Theorem 1. *For the map $\widehat{G} = X + H : \mathbb{C}^{19} \rightarrow \mathbb{C}^{19}$ defined in (4):*

- (i) *every nonzero component of H is homogeneous of degree three;*
- (ii) *$(JH)^{18} = 0$ and $(JH)^{17} \neq 0$;*
- (iii) *the three pairwise-distinct rational points listed in Appendix C have a common image.*

Consequently $\det J\widehat{G} \equiv 1$ and $\widehat{G}$ is not injective. In particular, the Jacobian Conjecture fails in the Yagzhev class already in dimension 19.

Proof. The three assertions are exact finite identities over $\mathbb{Q}$. Both accompanying verifiers check cubic homogeneity and the collision directly. They also compute the polynomial matrix powers and obtain

$$(JH)^{18} = 0,$$

whereas the $(4, 19)$ entry of the preceding power is

$$((JH)^{17})_{4,19} = -18t^{16}x^{10}y^6z^2 \neq 0. \quad (5)$$

Matrix indices in this note are one-based; the verifiers check the corresponding zero-based Python entry $(3, 18)$. Lemma 1 gives $\det J\hat{G} \equiv 1$, and the collision gives noninjectivity. $\square$

5 Exact verification and reproducibility

The manuscript, canonical map, verifiers, generated appendices, checksums, licenses, and versioned release archive are available from the project page

<https://harrischan.com/docs/yagzhev-19/>

The principal release files are linked directly at

<https://harrischan.com/docs/yagzhev-19/yagzhev19-note-v06.pdf>
<https://harrischan.com/docs/yagzhev-19/yagzhev19-release-v0.6.zip>

The canonical filenames inside the release are

`yagzhev19_map.json`, `verify_yagzhev19_sympy.py`, `verify_yagzhev19_stdlib.py`.

The release was tested with Python 3.13.5 and SymPy 1.14.0. From the release directory, run

```
python3 verify_yagzhev19_sympy.py
python3 verify_yagzhev19_stdlib.py yagzhev19_map.json.
```

The release contains a canonical JSON statement of H , the collision points, and their common image. With whitespace and key ordering fixed as distributed, its SHA-256 digest is

`f88a0883ce9509339c53a6a721390053b424039a8dd6c41ebc7770aeb955cccd`.

The verifier-script digests are

`82a169f01211110d8c55bbc03fe18f45c57a9b4bdebaeb07584851d8584baf15` (SymPy),
`9ab62f3ccc3b149e9b72676399173eb5a8b1f61ee36bd0107ecabf2c1e8c5814` (standard library).

Two verifiers use this object in complementary ways.

1. The SymPy verifier starts from the displayed 11-variable map, computes $L^{-1}\Phi$, extracts Q and C , constructs (4), and performs exact symbolic checks. Polynomial matrix multiplication is implemented sparsely. It also generates Appendices A–C from the checked expressions and verifies that the distributed `yagzhev19_appendices.tex` matches byte for byte.
2. The second verifier imports no computer-algebra package. It reads the canonical JSON, asserts the exact variable-order list, and represents rational polynomials as dictionaries of exponent vectors with coefficients in Python’s `fractions.Fraction`. It independently checks homogeneity, collision, (5), and $(JH)^{18} = 0$.

The distributed `SHA256SUMS` file freezes hashes for the JSON, both verifier scripts, the generated appendix, manuscript source, and compiled PDF. Code is released under the MIT License; the manuscript and canonical map are released under CC BY 4.0, as recorded in `LICENSE.md`. The theorem can therefore be checked directly from the canonical object without relying on the historical reduction pipeline or on numerical sampling.

6 Dimension comparison and future compression

For a normalized degree-three map on $\mathbb{C}^n$ with r nonzero cubic components, Proposition 1 produces a Yagzhev map in dimension $n + r + 1$. The publicly available constructions discussed above give:

Degree-three input	n	r	sparse lift $n + r + 1$
Demirci’s generic reduction [5]	39	12	52
u/Distinct-Soft-3991 [6]	17	8	26
Atkins-Turkish map [7]	11	7	19

Demirci’s published construction [5] uses all 39 auxiliary coordinates and therefore has dimension 79; the first row records what the same input gives after omitting coordinates corresponding to zero cubic components.

For the 11-variable map posted by Atkins-Turkish, the seven polynomials C_i with $i \in S$ are linearly independent. Hence an invertible linear recombination of the normalized target coordinates cannot reduce the dimension of their span below seven. Thus, for this fixed normalized input and this sparse-lift mechanism, linear recombination alone cannot reduce the seven required auxiliary coordinates. An improvement within this framework must therefore use a smaller or structurally different degree-three input, or a genuinely different homogenization mechanism. The bilinear factorization model of Lou [3] is a natural candidate for such future work, but it is not used in the present construction.

Contributions and acknowledgments

Levent Alpöge publicly announced the original three-variable map and credited Claude Fable 5 with producing it [1]. Andy Jiang and Aaron Lou gave geometric and factorization-based descriptions; Bartosz Naskrecki synthesized these viewpoints and independently audited the coordinate identifications [2, 3, 4]. Ali Demirci publicly posted a 79-dimensional Yagzhev realization [5]. The Reddit user u/Distinct-Soft-3991 posted a 17-variable degree-three reduction [6]. Joe Atkins-Turkish posted the factor-aware 11-variable map used here; his Gist states that its explicit construction, simplification, and verification code were generated by ChatGPT [7].

Harris Chan formulated the sparse homogenization, directed the symbolic construction and independent verification, audited the generated artifacts, and prepared the exposition, with assistance from Claude Fable 5, GPT-5.6 Sol, and Gemini 3.1 Pro. These systems are acknowledged as computational tools rather than authors; the author takes responsibility for all mathematical claims, citations, attribution decisions, and conclusions in this note.

A The 11-variable map

With coordinate order $(x, y, z, a, b, c, d, q, s, h, k)$, the components are

$$\begin{aligned}
\Phi_1 &= -ac - adz - 3ay^2 - 2az - cd^2 + d^2z - ds + 7dy^2 + sxy + 3xyz + 4y^2 + z, \\
\Phi_2 &= -bc - bdz - 3by^2 - 2bz - 3cdx - dq + qxy + 12xy^2 + 3xz + y, \\
\Phi_3 &= -hk - hxz + kx^2 - 3x^2y + 2x, \\
\Phi_4 &= a - d^2 + 2dxy, \\
\Phi_5 &= b + 3x^2y,
\end{aligned}$$

$$\begin{aligned}
\Phi_6 &= c + xyz + 3y^2 + 2z, \\
\Phi_7 &= d - xy, \\
\Phi_8 &= bz + 3cx + q, \\
\Phi_9 &= az + cd + cxy - dz + s - xyz - 7y^2, \\
\Phi_{10} &= h - x^2, \\
\Phi_{11} &= k + xz.
\end{aligned}$$

B Quadratic and cubic parts after normalization

The nonzero components of Q are

$$\begin{aligned}
Q_1 &= -\frac{hk}{2}, \\
Q_2 &= -bc - 2bz - dq + 3xz, \\
Q_3 &= -ac - 2az - ds + 4y^2, \\
Q_4 &= -d^2, \\
Q_6 &= 2ac + 4az + 2ds - 5y^2, \\
Q_7 &= -xy, \\
Q_8 &= bz + 3cx, \\
Q_9 &= az + cd - dz - 7y^2, \\
Q_{10} &= -x^2, \\
Q_{11} &= xz.
\end{aligned}$$

All omitted Q_i are zero. The nonzero components of C are

$$\begin{aligned}
C_1 &= -\frac{hxz}{2} + \frac{kx^2}{2} - \frac{3x^2y}{2}, \\
C_2 &= -bdz - 3by^2 - 3cdx + qxy + 12xy^2, \\
C_3 &= -adz - 3ay^2 - cd^2 + d^2z + 7dy^2 + sxy + 3xyz, \\
C_4 &= 2dxy, \\
C_5 &= 3x^2y, \\
C_6 &= 2adz + 6ay^2 + 2cd^2 - 2d^2z - 14dy^2 - 2sxy - 5xyz, \\
C_9 &= cxy - xyz.
\end{aligned}$$

C The lifted collision points

The lifted points are $\hat{p}_i = (p_i, -C_S(p_i), 1)$. In the coordinate order

$$(x, y, z, a, b, c, d, q, s, h, k, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_9, t),$$

they are

$$\hat{p}_1 = (0, 0, -\frac{1}{4}, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1),$$

$$\begin{aligned}\hat{p}_2 &= (1, -\frac{3}{2}, \frac{13}{2}, -\frac{9}{4}, \frac{9}{2}, -10, -\frac{3}{2}, \frac{3}{4}, -\frac{153}{8}, 1, -\frac{13}{2}, \\ &\quad \frac{17}{4}, \frac{45}{8}, -\frac{99}{16}, -\frac{9}{2}, \frac{9}{2}, \frac{177}{8}, -\frac{99}{4}, 1), \\ \hat{p}_3 &= (-1, \frac{3}{2}, \frac{13}{2}, -\frac{9}{4}, -\frac{9}{2}, -10, -\frac{3}{2}, -\frac{3}{4}, -\frac{153}{8}, 1, \frac{13}{2}, \\ &\quad -\frac{17}{4}, -\frac{45}{8}, -\frac{99}{16}, -\frac{9}{2}, -\frac{9}{2}, \frac{177}{8}, -\frac{99}{4}, 1).\end{aligned}$$

Their common image is

$$(0, 0, -\frac{1}{4}, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1).$$

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